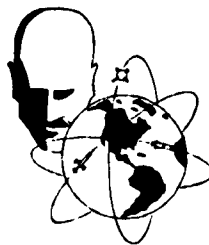ESD-TDR-63-155

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# REDUCTION OF SIDE LOBES AND POINTING ERRORS IN PHASED ARRAY RADARS BY RANDOMIZING QUANTIZATION STEPS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-155

April 1963

DIRECTORATE OF APPLIED RESEARCH

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

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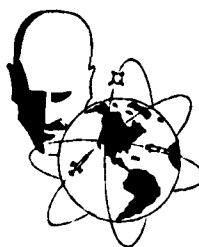Do not return this copy.   Retain or destroy.

# REDUCTION OF SIDE LOBES AND POINTING ERRORS IN PHASED ARRAY RADARS BY RANDOMIZING QUANTIZATION STEPS

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# ABSTRACT

In many array radars the composite beam is directed
by quantized phase shifters.  Regular sequence of quan-
tization errors enhances sidelobes and pointing errors.
It is recommended to break up the regularity by small,
known nonlinear and preferably quasi-random initial
phase deviations in the array elements.  This reduces
the lobes and pointing errors for a given number of
quantization steps, especially in high gain, multi-
element arrays.

Rapid electronic beam shifting of phased arrays can be carried out by continuous or "analog" phasing, for instance by ferrite phase shifters in which the phase angle increases monotonically with magnetizing current. However, this requires difficult programming.

Therefore many designs call for "quantized" phase shifters. These are usually cascaded bi-stable devices that introduce fixed amounts of delay or of phase shift, equal to $2\pi$ radians, divided by powers of 2.

With $n$ binary phasers, the phase step is $\Delta\phi = 2\pi.2^{-n} = \pi.2^{1-n}$    (1)

The maximum error is $\frac{1}{2}$ step, or $\varepsilon = 2^{-n}\pi$    (2)

Assuming that all antenna elements are designed for equal spacing, equal amplitude and, at a reference angle, zero phase in all elements (that is, no space nor amplitude taper) one may interpret the phase steps as error signals, approximately in phase quadrature, and with amplitudes $\varepsilon$.

The relative error power is $P = \varepsilon^2$. Since there is equal probability (3) of all error amplitudes between zero and $\varepsilon$, the mean error power is

$$\overline{P} = \varepsilon^2 \int_0^1 x^2 dx = \frac{1}{3}\varepsilon^2 \qquad (4)$$

This note will show to what extent the effects of these error components are enhanced by regularity and reduced by randomness.

## 1. Sidelobes

### 1.1 Linear Increase of Phase Shift with Element Number

If all antenna elements are correctly phased for a reference angle, and all phase shifters introduce equal phase delay in the "off" position, the quantization error approximates a sawtooth function. It increases linearly from zero to a maximum of about $+\varepsilon$, then takes a downward jump to approximately $-\varepsilon$, resumes its linear rise to $+\varepsilon$, and so forth.

Fourier analysis resolves the sawtooth function into a fundamental sine wave and its weak harmonic overtones. The amplitude of the $m^{th}$ harmonic component is

$$a_m = \frac{\varepsilon}{\pi^2} \int_{-\pi}^{\pi} x \sin mx\, dx = \frac{-2\varepsilon}{\pi m}(-1)^m \qquad (1-1)$$

The predominant fundamental is a sinusoidal phase modulation with the amplitude,

$$A_1 = \frac{2\varepsilon}{\pi} \qquad (1-2)$$

This generates two sidelobes with amplitudes

$$E_S = \frac{\varepsilon}{\pi} = 2^{-n} \qquad (1-3)$$

The width of these sidelobes equals that of the main lobe. The relative sidelobe level is

$$\Delta W = -6n \text{ db} \qquad (1-4)$$

For  5  quantization bits, the maximum error is

$$\ell = \frac{\pi}{32} = 5.625° \qquad (1-5)$$

and the relative sidelobe level due to quantization is

$$\Delta W = -30db \qquad (1-6)$$

## 1.2    Random Distribution of Phase Error

### 1.2.1 Linear Array

Let the phase of each element, at the reference (broadside) target aspect, differ from zero by a known angle that varies in a quasi-random manner between the limits  0  and  $\ell$.  This requires a different programming of the phase shifters and breaks up the regularity of the sawtooth pattern.  The result is a noiselike phase modulation with a power per equation (4).

This power is randomly distributed over the entire scanning angle  $\theta$  while that of the main beam is concentrated in the beam with angle  $\beta$. The probability of noise power exceeding its mean value by a factor  k  is

$$P_{(k)} = \epsilon^{-k^2} \qquad (1-7)$$

Hence a peak factor not to be exceeded more than 0.2% of the time is

$$k = \sqrt{\log_\epsilon 500} = 2.5 = +8db \qquad (1-8)$$

If one assumes that the antenna elements limit the angular spread of random error signals to the scanning angle  $\theta$,  while the main beam is contained within its beam width  $\beta$,  one finds the relative level of the highest random noise lobe as  ·

$$\Delta = +8 + 10 \log \left(\frac{\overline{P}}{1-\overline{P}} \frac{\beta}{\theta}\right) db \qquad (1-9)$$

From (1-9) and (4)

$$\Delta \doteq 13.2 - 6n - 10 \log \left(\frac{\theta}{\beta}\right) db, \qquad (1-10)$$

provided  $P \ll 1$.

For a beam width of 1° and a scanning angle of 90°

$$\Delta = 13.2 - 19.5 = -6.3 - 6n \qquad (1-11)$$

Hence a 30db peak sidelobe level can be obtained with 4 quantization steps instead of 5.

### 1.2.2 Two-dimensional Array

In this case the scan and beam areas are solid angles.  Equation (1-10) results in too low a number of quantum steps.  One must use the

exact equation (1-9) which includes the reduction of main beam energy by sidelobe energy.

Setting $\Delta = 30$ in equation (1-9) one finds

$$10 \log \left( \frac{\overline{P}}{1-\overline{P}} \frac{\beta}{\theta} \right) = -38 \qquad (1\text{-}12)$$

$$\frac{\overline{P}}{1-\overline{P}} = \frac{\theta}{\beta} \times \frac{1}{6250} \qquad (1\text{-}13)$$

For a needle beam of 1° width and a scan of 90° in azimuth and elevation,

$$\theta = \frac{\pi}{2}, \quad \beta = 3.10^{-4} \quad \text{and}$$

$$\frac{\overline{P}}{1-\overline{P}} = \frac{\pi}{6} \frac{10^4}{6250} = \frac{5}{6} \quad \text{or}$$

$$\overline{P} = \frac{5}{11} = 0.454$$

$$n \geq \frac{\log \frac{\pi^2}{3P}}{2 \log 2} \geq 1.44 = 2 \qquad (1\text{-}14)$$

This indicates that a quantum step of 90°, or a randomized phase error with a maximum of 45° is sufficient to insure a 30db random side lobe ratio in an array of 8100 elements.

Note. In order to be able to utilize a -30db sidelobe level, the unquantized sidelobes must be lower than -30db. This requires amplitude or space taper and correspondingly reduces the gain of the main beam and the effective number of randomized elements which enter the above calculation.

However, this does not invalidate the advantage of randomizing the size and polarity of quantization errors.

Before accepting as valid design a number of steps as low as that per Equation (1-14) one must check the systematic or random pointing error resulting from the phase steps.

## 2.    Pointing Error

### 2.1    Linear Increase of Phaseshift

Let the length of a linear array be $D$ and its wavelength, $\lambda$. Then its beamwidth is

$$\beta = \frac{\lambda}{D} \qquad (2\text{-}1)$$

A phase error $\pm \varepsilon$ in any element corresponds to a longitudinal displacement of

$$\hat{y} = \frac{\varepsilon \lambda}{\pi} = \lambda \, 2^{-(n+1)} \qquad (2\text{-}2)$$

It is assumed that the center element of the array remains at zero phase, and that the phaseshift of the $(-i)^{th}$ element is equal and opposite to that of the $(+i)^{th}$ element.

If the target angle from the normal is equal to or smaller than the threshold angle

$$\alpha_{th} \leq 2^{-n} \beta, \qquad\qquad (2\text{-}3)$$

all required phase shifts are $\leq \ell$ and no phase shift takes place. Hence the maximum pointing error is

$$\gamma_{max} = 2^{-n} \beta = \alpha_{th} \qquad \text{(see Figure 1)} \qquad (2\text{-}4)$$

When the target angle approximates an even multiple of $\alpha_{th}$,

$$\alpha \doteq 2m\alpha_{th} \qquad \text{(see Figure 2)} \qquad (2\text{-}5)$$

By least square methods one finds the pointing error

$$\gamma \doteq \frac{-\alpha th}{4m} \qquad\qquad (2\text{-}6)$$

When the target angle approaches an odd multiple of $\alpha_{th}$

$$\alpha = (2m + 1)\alpha_{th} \qquad \text{(see Figure 3)} \qquad (2\text{-}7)$$

and the pointing error

$$\gamma \doteq \frac{\alpha th}{2m+1} \qquad\qquad (2\text{-}8)$$

## 2.2    Randomized Phaseshift

Let an array have $N$ elements equally spaced over its length $D$. Let the $i^{th}$ element have a phase error $\epsilon_i$. The mean phase error is

$$\bar{\epsilon} = \frac{1}{2} \ell = \pi \cdot 2^{-(n+1)} \qquad\qquad (2\text{-}9)$$

Disregarding any constant phase shift components, one finds the least square fit for a linear phase slant by making

$$\sum_{i=-\frac{N}{2}}^{N/2} (\epsilon_i - \frac{2xi}{N})^2 = min \qquad\qquad (2\text{-}10)$$

For large $N$ this approaches the solution

$$\sum_i (i\epsilon_i - \frac{2}{N} xi^2) = 0 \qquad\qquad (2\text{-}11)$$

$$\sum_i i\epsilon_i = \sqrt{\bar{\epsilon^2} \, \epsilon^2} = \bar{\epsilon}_i \sqrt{\frac{N^3}{12}} \qquad\qquad (2\text{-}12)$$

$$\sum_i \frac{2}{N} \times i^2 = x \frac{N^2}{12} \qquad\qquad (2\text{-}13)$$

By equating (2-12) to (2-13) one finds the rms shift at the edges of the

array:

$$|\bar{x}| = \tau \sqrt{\frac{12}{N}} = \ell \sqrt{\frac{3}{N}} = 2^{-n}\pi \sqrt{\frac{3}{N}} \qquad (2\text{-}14)$$

The equivalent displacement is

$$\bar{y} = \frac{\bar{x}\lambda}{2\pi} = 2^{-(n+1)} \lambda \sqrt{\frac{3}{N}} \qquad (2\text{-}15)$$

and the rms pointing error

$$\bar{\gamma} = \frac{2\bar{y}}{D} = 2^{-n} \beta \sqrt{\frac{3}{N}} \qquad (2\text{-}16)$$

Assuming an 8db random peak factor as in Equation (1-12), one finds

$$\hat{\gamma} = \frac{4.3}{\sqrt{N}} 2^{-n} \beta \qquad (2\text{-}17)$$

for the largest random pointing error.

When the number of array elements (N) exceeds 100, randomization reduces the peak pointing error greatly, as seen by comparing Equations (2-4) and (2-17).

## Conclusion

A small quasi-random but known variation in the initial phase of the array elements breaks up the regular, repetitive pattern of quantization steps and thus reduces the sidelobes and the maximum pointing error of the beam.

Note: The regularity of quantization steps is also somewhat reduced if the initial phases of the elements are adjusted according to some non-linear analytic functions.
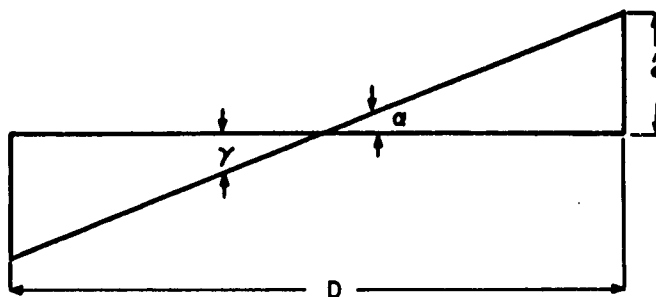
W. J. Albersheim

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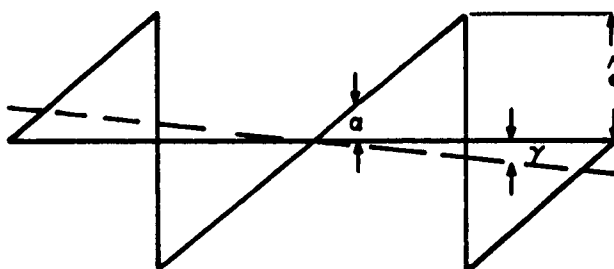
Figures 1, 2, 3 (1 sheet)

## LEAST SQUARE STRAIGHT LINE FIT TO
## SAWTOOTH PHASE QUANTIZATION ERRORS

TARGET ANGLE $\alpha = \alpha_{TH} = 2^{-N}\beta$     POINTING ERROR $\gamma_{MAX} = \alpha_{TH}$
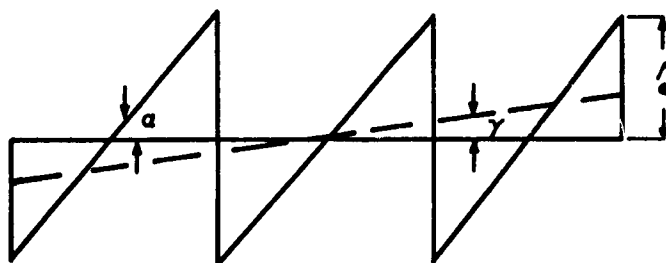
FIG I

SINGLE SAWTOOTH

TAN $\alpha = 2M$ TAN $\alpha_{TH}$     $\alpha = 2_M \alpha_{TH}$     $\gamma = -\dfrac{\alpha_{TH}}{4M}$

FIG 2

EVEN NUMBER OF SAWTEETH

TAN $\alpha = (2M+1)$ TAN $\alpha_{TH}$     $\alpha = (2M+1 \alpha_{TH}$     $\gamma = +\dfrac{\alpha_{TH}}{2M+1}$

FIG 3

ODD NUMBER OF SAWTEETH

I – I

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